Template-based treemaps to preserve spatial constraints

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Abstract:
Treemapping is a method for displaying hierarchical data using nested rectangles. Each branch of
the tree is given its rectangle, which then is tiled with smaller rectangles representing sub-branches.
A node’s shape has an area proportional to a specified dimension of the data. To create a treemap,
one must define a tiling algorithm, i.e., a way to divide a rectangle into sub-rectangles of specified
areas. There are several problems with existing tiling algorithms: (i) they allow tiles to shift when
the main window or some of the tiles are resized affecting the user perception of the information;
(ii) they do not allow users to place selected elements into desired positions w.r.t. each other.
In this paper, we present a method for creating treemaps with customized layouts. The method
is based on reusable templates and supports zooming into specific areas of the treemap without
affecting its initial layout. We illustrate the use of template-based treemaps to the visualization
of biomedical data. Furthermore, we present an algorithm for automated generation of layouts
satisfying positional constraints for a particular class of constraints.

1 Introduction

Treemaps (Johnson and Shneiderman, 1991) are an effective technique to visualize two-
dimensional hierarchical data by using nested shapes in a space-filling layout. The standard
shape is the rectangle, but other shapes and variations, e.g., circular treemaps (Fischer et al., 2012;
Wetzel, 2006) and Voronoy diagram-based subdivisions (Balzer and Deussen, 2005), have been
used. Each shape represents a geometric region, which can be subdivided recursively into smaller
regions. At each level of recursion, some attribute of interest is displayed, e.g., with the help of shape
color.

Nodes in a treemap, also called \textit{tiles}, represent individual data items in a dataset. Node size,
color and text label can be used to represent attributes of the data item. One-layered treemaps
can display data attributes but are not very good at emphasizing the place of an item in the overall
hierarchical structure. To compensate for that, a small fixed margin with structural labels are typ-
ically used. In treemaps displaying hierarchical structures, it is possible to navigate among dif-
ferent layers and zoom into selected tiles (Blanch and Lecolinet, 2007). A more complete overview
of treemap development history and applications can be found in (Shneiderman, 2009).

To create a treemap, one must define a tiling algorithm - a way to divide a rectangle into sub-
rectangles of specified areas. In most tiling algo-
rithms, the node position does not precisely repre-
sent any characteristic of the data. This can
reduce the clarity of the representation by viol-
ating user expectations to locate certain item
in a certain place. The same data can be rep-
resented in arbitrarily different looking treemaps
depending on the treemap dimensions and order-
ning of nodes. Even if the same tiling algorithm is
used for the identical datasets, their visual rep-
resentations may vary drastically. The ability of
the tiling algorithm to preserve locations of indi-
vidual data items upon changes of visualization
and dataset parameters is referred to as \textit{stability}.
Most of the tiling algorithms target at optimizing
metrics other than stability. An average \textit{as-
pect ratio} (the longest side of a rectangle divided
by its shortest side), and whether the natural or-
der of the dataset is preserved in its visualization are two important characteristics (Wattenberg, 2005).

Among the most commonly used tiling algorithms are slice and dice (Johnson and Shneiderman, 1991), squarify (Bruls et al., 2000), strip (Bederson et al., 2002), and their numerous variations. These algorithms do not guarantee the complete adherence to two-dimensional spatial constraints. For example, Figure 1 shows how the representation of the same hierarchical structure using the squarify algorithm (Bruls et al., 2000) changes when the proportions of the display area are changed.

The need for a tiling algorithm that allows users to control treemap layouts has been pointed out in (de Bono et al., 2012). This work applies treemaps to automatically generate biomedical maps from publicly available data sources by overlaying the information about blood vessels, proteins, gene expressions, cells, drug and disease mechanisms, etc. over graphical schematics of human body anatomy. In these treemaps,

- it is expected that spatial relations and constraints on relative node positioning among body parts are always preserved,
- there should be a possibility to filter the dataset by showing only selected tiles of interest to a certain user without affecting the visual stability of the corresponding treemap.

The concepts of a constrained treemap layout and custom user templates have emerged, but no generic methods to build such templates have been developed.

In this paper, we address the problem of building customized and controlled treemap layouts. First, we describe our effort to control tile positions using a constraint solver (Section 2). Then we introduce a concept of a template: a hierarchical structure that allows users to pose constraints on relative tile positioning (Section 3). We present a graphical tool to create such templates and a tool to draw treemaps with customized layouts by binding such templates with treemap datasets using an example from the biomedical domain (Section 4). Furthermore, we introduce a problem of automated generation of templates to satisfy spatial constraints and provide an algorithm to generate such templates for a particular class of constraints (Section 5). Finally, we provide an overview of the state of the art on data visualization using treemaps (Section 6), conclude the paper and discuss future work (Section 7).

## 2 Constraining treemap layouts

The work on treemap generalization (de Bono et al., 2012; Tsiaras et al., 2009) inspired us to apply a constraint solver to build layouts for a general set of geometric constraints. We encoded the conditions for a desired treemap layout and employed Z3 SMT solver (de Moura and Björner, 2008) to find solutions for data with constraints on the aspect ratio of treemap tiles. As a tiling algorithm is typically a recursive procedure, it is sufficient to show how tiles are allocated on one level. Our general approach is outlined in Figure 1. This listing shows a pseudocode for a constraint satisfaction problem (CSP) that for a given display area defined by its left top corner coordinates and dimensions and a list of \( n \) elements that should be placed in this area returns positions and dimensions of treemap rectangles.

The constraints define bounds on element positions and dimensions, and require that any two rectangles should not intersect. The sum of areas of all rectangles should be equal to the main area. Finally, any custom constraints on element positions, dimensions, adjacency etc. can be defined.

We applied the presented algorithm to find a treemap layout for a simple tree consisting of a root node with three children elements. A custom constraint here is a requirement that the width of rectangles should be larger than their height. The solution for the CSP with no custom constraints is shown in 2(a). It is found in less than a second, while to find a solution for the layout with the additional constraint (Figure 2(b)) the solver needed several minutes. Thus, while it is feasible to build treemap layouts with any desired properties using a constraint solver, the approach is too slow for ad-hoc visualization. Even in trivial cases, the SMT solver may fail to satisfy the constraints due to their non-linear nature or large search space. The stability of the layouts is not guaranteed either.

From this we conclude that the search space for possible layouts should be restricted to some
geometrical pattern. For example, the strip (Bederson et al., 2002) algorithm allows only layouts composed of horizontal or vertical stripes at each level. Although this algorithm produces ordered treemaps, it is not sufficient for our goals due to its instability (elements may jump from one line to another when element sizes are changed) and inability to impose two-dimensional positioning constraints.

The template-based treemapping algorithm we propose uses the multi-level splitting of a given area into columns and rows. Two-dimensional constraints on positions of treemap elements are satisfied by placing such elements into areas corresponding to different branches of a template.

3 Defining templates

In this section, we describe our method to create templates for treemap tile allocation.

A template is defined in the form of a tree where each node has an assigned attribute to represent a vertical split or a horizontal split operation. Leaf nodes of the tree correspond to the treemap tiles and define their relative positions. Internal nodes of the tree define the way the available area is subdivided. Color or image can be assigned to each node of the template (typically, only appearance of leaf nodes matters). Additionally, one may want to use only part of the area allocated to the parent rectangle to place children elements. This can be accomplished with the help of vertical ratio and horizontal ratio parameters of a template node. If not the entire area of the tile is occupied, we can use vertical alignment and horizontal alignment properties with values top, middle, down and left, center, right, respectively, to position nested elements within the tile.

A layout defined by a template is build by splitting the available area at each level using the operation defined in template nodes. All the leaf tiles of the template that appear on the top level of such a treemap will always remain at the same relative positions to each other. Figure 3 shows our template builder featuring a sample template with 6 tiles. At the first level, the entire area is split into 2 columns. At the second level, the second column is split into 2 rows. At the third level, the bottom row is split into 2 columns and, finally, its first column is split into 3 rows. Figure 3(b) shows the rendering of the template.

The graphical editor prototype we used to create this template is available at http://nkokash.com/apinatomy. The defined templates can be exported to files and reconstructed from such files when bound to treemap

Algorithm 1: Generic constraint-based tiling algorithm

```plaintext
/*Define terms representing the position and dimensions of the main tile */
x0 ← 0; y0 ← 0; w0 ← area.width; h0 ← area.height;

/*Define arrays T, P, N, W of terms representing the positions and dimensions of its children, 0 < i ≤ n */
/*The dimensions of each child should not exceed the dimensions of the main rectangle */

for 0 < i ≤ n do
  assertCnstr ((x_i ≥ 0) ∧ (y_i ≥ 0) ∧ (w_i > 0) ∧ (h_i > 0));
end

/*Rectangles should not intersect */

for 0 < j ≤ n do
  assertCnstr (((x_j + w_j ≤ x_i) ∨ (x_i + w_i ≤ x_j)) ∧ ((y_j + h_j ≤ y_i) ∨ (y_i + h_i ≤ y_j)));
end

/*Define area constraints for each child rectangle */

for 0 < i ≤ n do assertCnstr ((h_i * w_i) = w0 * h0 * size/ totalSize);
end

/*The sum of areas of all rectangles should be equal to the main area */
totalArea ← 0;
for 0 < i ≤ n do totalArea ← totalArea + h_i * w_i;
assertCnstr (totalArea = w0 * h0);

/*Add custom constraints here, solve the CSP */
```
items. The number of leaf items in the template should coincide with the number of the children of the tile it is assigned to. The customized treemap can be stored and further reused without the need to reassign templates.

Algorithm 2 computes positions and dimensions of tiles representing children of each treemap item. Each slice is allocated the area proportional to the total area of all its items w.r.t. to the total area of all items in the parent rectangle. The allocation of the corresponding area is controlled either by the width or by the height of the strip while its other dimension remains unchanged at this level. The leaf nodes of the template tree define the layout for a single level of a treemap. In order to keep the algorithm simple, we omit the processing of scaling and alignment attributes.

4 Template-based treemaps

In this section, we illustrate the application of templates to control the positions of tiles in a treemap representing human anatomy.

Our example is explained in Figure 4 which depicts a longitudinal section through the middle of the male human body showing the external and internal surfaces, and organs. From this view, we obtain an idealized radially symmetric body plan, apportioned over cylindrical regions. Each of the organs in the plan is composed of multiple tissues and sub-organs, the structural information about them is obtained from the Foundational Model of Anatomy (FMA) ontology. As the visualization of the full ontology may obscure the details a user is interested to see, it is essential for the visualization tool to support data filtering across multiple levels and contextual zooming into selected areas. The user should be able to create a custom view with the internal structure of the selected body parts placed in a way that simplifies the analysis of these data.

We reproduce this view on human anatomy in our prototype. The first level of the “Body” treemap consists of 24 equal-size tiles. The template that controls this layout consists of 4 rows, each in its turn contains 6 columns. We illustrate how the template-based algorithm works by assigning the template created in the previous section to the item “6000002: Liver Pancreas Duodenum”. Its initial layout (tiles are placed in a grid) is shown in Figure 5(a). Figure 5(b) shows the allocation of the tile children according to the predefined template. Note that in contrast to the Figure 3(b) where we emphasized the splitting defined in the template, the treemap layout is formed to preserve the ratio among item sizes (all equal in this case). To enlarge a certain area of a template-based treemap, it is sufficient to update the size dimension of the item. Figure 5(c) demonstrates that the tile relative positions are preserved after tile resizing. Moreover, if one decides to filter the treemap to remove items that are not of interest for data analysis, the remaining tiles will occupy the available area and their relative positions will be preserved as well.

The “Body” treemap prototype can be seen at http://nkokash.com/apinatomy. Several views are available to illustrate the difference of our approach to the existing treemapping algorithms, which become apparent once one modifies the treemap by removing or including new items or resizing selected tiles. For example, Figure 5(d) shows two versions of the custom treemaps for the body anatomy. The left treemap includes 24 upper level tiles with some details about the structure of selected parts, including “6000002: Liver Pancreas Duodenum”. The right image shows a modification of this treemap where some tiles are removed and the sub-parts of others are shown in

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Input: TreemapItem template - reference to the template associated with a treemap item; Rect currRect - a rectangle defining the available area for this item; Output: List ⟨Rect⟩ finalRects - a list of tile dimensions for item children; List ⟨TreemapItem⟩ finalTiles - a list of tiles; void TemplateBased (TreemapItem template, Rect currRect, List ⟨Rect⟩ finalRects, List ⟨TreemapItem⟩ finalTiles)
begin
if template.Children.Count > 1 then
    float prevX = currRect.X, prevY = currRect.Y;
    foreach TreemapItem subTemplate in template.Children do
        float fraction = subTemplate.FinalSize / template.TotalChildrenSize;
        float dWidth = currRect.Width, dHeight = currRect.Height;
        if template.Split == "horizontal" then dHeight *= fraction else dWidth *= fraction;
        Rect rect = new Rect (prevX, prevY, dWidth, dHeight);
        TemplateBased (subTemplate, rect, finalRects, finalTiles);
        if template.Split == "horizontal" then prevY += dHeight else prevX += dWidth;
    end
else
    /*Leaf tile occupies all available area*/
    finalRects.add(currRect);
    finalTiles.add(template);
end
end

Algorithm 2: Template-based tiling algorithm

(a) Schematic body plan  (b) Visualizing medical ontologies using treemaps
Figure 4: Longitudinal section through the middle of the male human body
more detail. The overall layout remains stable.

Publicly available ontologies and other resources maintained by biomedical communities provide a rich source of information about human body which however is hard to perceive and process in the existing form. Treemapping templates we introduced in this paper provide a quick way to extend these data with the information about spacial representation of related concepts in a schematic way. Thus, our approach may find multiple applications in the area of biomedical data analysis, e.g., for the generation of teaching materials or reasoning about large amounts of interconnected data.

5 Template generation

We illustrated how a user can control treemap layout using manually defined templates. In this section, we consider the problem of generating such templates automatically.

Multiple variations in the nature and description of tile positioning constraints are possible. We introduce here a CSP which consists in finding a treemap layout satisfying requirements in the form “item $x$ should be placed left (right) and/or above (below) item $y$”. More formally, let $S$ be a list of elements with a given dimension $s.size, s \in S$. Assume also that for each $s \in S$, we generate rectangles identified by their top left corner coordinates, width and height. Thus, we can refer to $s.x, s.y, s.w$ and $s.h$ as to the $x$ and $y$ coordinates of the rectangle top left corner, its width and height, respectively. We write

$$s_j \rightarrow s_i, i \neq j \text{ iff } s_j.x + s_j.w \leq s_i.x$$

and

$$s_j \uparrow s_i, i \neq j \text{ iff } s_j.y + s_j.h \leq s_i.y;$$

Our goal is to generate a treemap layout that satisfies a system of constraints

$$C = \{s_iR s_j, s_i, s_j \in S, R \in \{\rightarrow, \uparrow}\}.$$

Easy to notice that a set of one-dimensional constraints, either $\{s_i \rightarrow s_j, s_i, s_j \in S\}$ or $\{s_i \uparrow s_j, s_i, s_j \in S\}$, can be satisfied by separating elements $s_i$ and $s_j$ into subsets and placing elements of each subset into a treemap region separated from the regions for other subsets by vertical or horizontal lines, respectively.

To refer to the transitive constraint satisfaction, we can write $s_j \Rightarrow s_i$ iff there exists $s_k \in S$ such that $s_j \rightarrow s_k$ and $s_k \rightarrow s_i$; Analogously, $s_j \nmid s_i$ iff there exists $s_k \in S$ such that $s_j \uparrow s_k$ and $s_k \uparrow s_i$. Clearly, the system of constraints is not satisfiable if there exist $s_i, s_j$ such that $s_i \Rightarrow s_j \Rightarrow s_i$ or $s_i \nmid s_j \nmid s_i$.

We say that a system of constraints

$$C = C_H \land C_V,$$

where

$$C_H = \{s_i \Rightarrow s_j, s_i, s_j \in S\}$$

and

$$C_V = \{s_i \nmid s_j, s_i, s_j \in S\}$$
orthogonal constraints, we dynamically build a set of items displayed as treemap tiles. In corresponding expressions) represent dataset ↑ satisfying → slice satisfying ↓ slice. An internal node of a slicing tree represents a vertical dataset with additional visual attributes. Every operator defining the type of the split operation can be represented using an infix expression with operands referring to free tile sets or sub-trees and be represented using an infix expression with slicing tree 2002; Lai and Wong, 2001). A looking for a suitable layout by constructing and rendering slicing trees as in the previous case. For brevity, we use figure brackets {...} to refer to all possible two-dimensional permutations of elements we do not need to unfold at a particular step. Figure 7 shows a slicing tree with the first constraint satisfied and the corresponding treemap layout. Infix expressions can be used to describe both initial constraints and the slicing trees at any intermediate step:

\[
c_1 = s_1 \rightarrow \{s_2, s_3\}, \quad c_3 = s_6 \uparrow s_1, \\
c_2 = s_5 \rightarrow s_6, \quad c_4 = s_3 \uparrow s_5, \\
c_5 = s_2 \uparrow s_4.
\]

The first set of slicing trees \(T_1\) is equivalent to \(c_1\). Next, we join a constraint \(c_2\). Since none of its items is in \(T_1\), this case folds into case 4 of the

Figure 7: Building and rendering slicing trees

is orthogonal iff for any \(s_i, s_j\) such that

\[(s_i \Rightarrow s_j) \in CH\]

it follows that

\[(s_i \parallel s_j) \notin CV.\]

The orthogonality of vertical and horizontal constraints suggest that they can be satisfied in a template obtained by dividing the area into sub-regions using vertical and horizontal splits. The satisfaction of non-orthogonal systems of constraints is problematic since we are not able to guarantee the existence of the solution in the form of the rectangular treemap without gaps. Figure 6 shows two possible layouts for a dataset with two elements with non-orthogonal constraints. Such constraints require elements to be placed diagonally and remaining empty areas to be filled with other tiles. It is easy to see that there may be no items of suitable areas in the dataset to fill these areas without gaps. An appropriate layout, if it exists, can be searched for in the form of a CSP as described in Section 2.

For a dataset with orthogonal constraints, we look for a suitable layout by constructing and evaluating all possible slicing trees (Chen et al., 2002; Lai and Wong, 2001). A slicing tree can be represented using an infix expression with operands referring to free tile sets or sub-trees and an operator defining the type of the split operation. The definition of a slicing tree essentially coincides with the definition of our template: a template can be seen as a slicing tree for the complete dataset with additional visual attributes. Every internal node of a slicing tree represents a vertical slice satisfying → constraint or a horizontal slice satisfying ↑ constraint. Leaf elements (or atoms in corresponding expressions) represent dataset items displayed as treemap tiles.

To construct a template for a given set of orthogonal constraints, we dynamically build a set of trees that satisfy constraints one by one (Algorithm 5). Thus, given a constraint \(c = s_i R s_j\), we check whether items \(s_i\) and \(s_j\) are already in the set of slicing trees obtained at the previous step. If both items are found, we check whether the constraint \(c\) is satisfied. Assume that function search(Tree tree, Item s) returns a path from the item \(s\) to the tree root. Using this function, we find paths \(path1\) and \(path2\) from items \(s_i\) and \(s_j\) to the tree root. The constraint \(c\) is satisfied if the operator of the node where both paths deviate coincides with the operator in the constraint and the left handed item \(s_i\) is located to the left from the deviation point, that is, the next node of the \(path1\) is the left child of the node at the deviation point.

If only one of the items is in the partial tree (say, \(s_i\)), we can insert the second item \(s_j\) in such a way that the constraint \(c\) is satisfied. This can be achieved either by inserting the missing item in any branch of the tree that is already separated from the existing item using the splitting operator in \(c\), or by creating a new split at the root of the tree or right to the existing item. If none of the items in the constraint are in the tree, we create all the permutations containing one of the items and then add the second item to each of these trees as in the previous case.

Let us consider an example. Assume we have a dataset

\[S = \{s_i, 1 \leq i \leq 6\}\]

with the following sets of orthogonal constraints

\[CH = \{s_1 \rightarrow s_2, s_1 \rightarrow s_3, s_5 \rightarrow s_6\}\]

and

\[CV = \{s_2 \uparrow s_4, s_3 \uparrow s_5, s_6 \uparrow s_1\}.
\]

Figure 6: Satisfying non-orthogonal constraints
Input: List<Constraint> constraints - a list of constraints;
Output: List<Tree> slicingTrees - a list of slicing trees satisfying constraints;
List<Tree> defineLayout(List<Constraint> constraints)
begin
    List<Tree> slicingTrees = new List<Tree> (new Tree (constraints[0]));
    constraints.remove(0);
    foreach c in constraints do
        List<Tree> newSlicingTrees = new List<Tree>();
        foreach tree in slicingTrees do
            newSlicingTrees.addAll(generateTrees(tree, c));
        slicingTrees = newSlicingTrees;
    end
end
List<Tree> generateTrees(Tree tree, Constraint c)
begin
    List<Constraint> path1 = search(tree, c.left);
    List<Constraint> path2 = search(tree, c.right);
    /* 1. Both items are already in the tree */
    if (path1 != null) and (path2 != null) then
        int i = 0;
        while path1[path1.length - 1 - i] == path2[path2.length - 1 - i] do i++;
        Constraint constraint = path1[path1.length - i];
        /* If the operator in this node coincides with the operator in the constraint and the rest of path1 is in its left branch, the constraint is satisfied. Otherwise, the constraint is violated and the slicing tree is eliminated. */
        if (constraint == c.operator) and (constraint == path1[path1.length - 1 - i]) then return tree; else return null;
    end
    List<Tree> slicingTrees = new List<Tree> ();
    /* 2. Left item is in the tree */
    if (path1 != null) and (path2 == null) then
        if path1[0] == c.left then
            foreach node in path1 do
                /* for all ancestors with the same split type located on the same side with the existing item */
                if (node.operator == c.operator) and (node.parent.left == node) then slicingTrees.addAll(insert(node.right, c.right));
            else
                /* analogously, but insert the missing element into the left branch of each ancestor with the same operator that has the existing item on the right */
                slicingTrees.add(new Tree (tree, c.constraint, c.right));
            end
        end
    end
    /* 3. Right item is in the tree */
    if (path1 == null) and (path2 != null) then
        /* analogously to the case 2 */
    end
    /* 4. None of the items is in the tree */
    if (path1 == null) and (path2 == null) then
        List<Tree> newSlicingTrees = insert (tree, c.left);
        foreach tree in newSlicingTrees do
            /* analogously to the case 2 */
        end
        return slicingTrees;
    end
end
List<Tree> insert(Tree tree, Constraint c)
begin
    List<Tree> slicingTrees = new List<Tree> ();
    slicingTrees.addAll(new Tree (tree, Operator.uparrow, c));
    slicingTrees.addAll(new Tree (tree, Operator.rightarrow, c));
    if tree.right != null then
        slicingTrees.addAll(tree.right, c);
    end
    if tree.left != null then
        slicingTrees.addAll(tree.left, c);
    end
    return slicingTrees;
end
Algorithm 3: Template generation algorithm
algorithm, i.e., all permutations are generated:

\[
\begin{align*}
T_2 &= T_1 \triangleright c_2: \\
T_3 &= T_2 \triangleright c_3: \\
T_4 &= T_3 \triangleright c_4 = (s_5 \rightarrow s_6) \cup (s_1 \rightarrow \{s_2, s_3\}).
\end{align*}
\]

Note that the positions of both items from \(c_3\) are predefined in the generated trees. Thus, at this step the case 1 of the algorithm is applied and we eliminate trees that do not satisfy \(c_3\), leaving options \(T_2^1\) and \(T_2^2\), where \(T_2^2\) is further unfolded and filtered to leave only trees satisfying \(c_3\):

\[
\begin{align*}
T_3^1 &= (s_5 \rightarrow s_6) \cup (s_1 \rightarrow \{s_2, s_3\}), \\
T_3^2 &= \left((s_5 \rightarrow s_6) \cup s_1\right) \rightarrow \{s_2, s_3\}, \\
T_3^3 &= (s_5 \rightarrow (s_6 \uparrow s_1)) \rightarrow \{s_2, s_3\}.
\end{align*}
\]

The items of constraint \(c_4\) are already positioned, so we again filter the set \(T_3\) according to the option 1 of the algorithm, leaving \(T_3^1\) as the only valid alternative:

\[
T_4 = T_3 \triangleright c_4 = (s_5 \rightarrow s_6) \cup (s_1 \rightarrow \{s_2, s_3\}).
\]

After joining the remaining constraint \(c_5 = s_2 \uparrow s_4\) according to the case 3 (\(s_2\) is in the tree \(T_4\) while \(s_4\) is a new item), we obtain the following set of slicing trees:

\[
\begin{align*}
T_5 &= T_4 \triangleright c_5 : \\
T_5^1 &= (s_5 \rightarrow s_6) \cup \left((s_1 \rightarrow \{s_2, s_3\}\uparrow s_4\right), \\
T_5^2 &= (s_5 \rightarrow s_6) \cup \left(s_1 \rightarrow \{s_2, s_3\}\uparrow s_4\right), \\
T_5^3 &= (s_5 \rightarrow s_6) \cup (s_1 \rightarrow \{s_2 \uparrow s_4, s_3\}).
\end{align*}
\]

If there are items in the data set without positional constraints, they can be inserted to any place in the final slicing trees. Together with two-dimensional permutations of elements \(\{s_2, s_3\}\) in the generated solutions, 12 layouts shown in Figure 8 satisfy our initial constraints. In total we get 11 unique layouts as layouts 2(d) and 3(a) in Figure 8 represent identical treemaps because permutations of elements we did not unfold in slicing trees \(T_2^2\) and \(T_2^3\) yield the same layouts.

Assuming that we target at finding all acceptable layouts, in the worst case we need to generate all two-dimensional permutations of the given set of tiles. Having in mind that to generate such permutations each of the items is placed to the left and right branches of the slicing tree for both vertical and horizontal split operators, the following dynamic formula applies \(F(n) = 4 \times n \times F(n - 1)\) and the algorithm’s performance can be estimated as \(\Theta(4^n \times n!)\). However, in practice we need only one suitable layout, and if no specific constraints on the tile positions are given, it is defined in \(\Theta(n)\). In the case of constrained layouts, each new constrained item needs to be added to the slicing trees generated at the previous step and this may lead to the generation of two-dimensional permutations of the current element with all previously inserted items. Thus, the complexity can be estimated as \(\Theta(4^n \times m!)\), where \(m\) is the number of unique data items with constrained positions. A more detailed analysis of the algorithm’s performance as well as its possible optimizations will be studied in future work.

6 Related work

In this section, we overview related work and compare our method to existing approaches.

Various extensions and modifications of classic algorithms exist that target at improving treemap performance metrics. Spatially ordered squarified layout (Wood and Dykes, 2008) provides a more consistent arrangement of nodes w.r.t. the simple unordered squarified layout while maintaining low aspect ratios. Buchin et al. (Buchin et al., 2011) describe algorithms for transforming a rectangular layout without hierarchical structure, together with a clustering of the rectangles of the layout, into a spatial treemap that respects the clustering and, to the extent possible, the adjacency of the input layout. Berg et al. (de Berg et al., 2011) study tiling algorithms with bounded aspect ratios. They show that one cannot bound the aspect ratio if the regions are restricted to be rectangles. Tak and Cockburn (Tak and Cockburn, 2013) introduce a new metric to capture more efficiently the notion of spatial stability and

![Figure 8: Automatically generated constraint-satisfying layouts](image-url)
introduce tiling algorithms designed to perform well according to this metric. We find our method complimentary to such algorithms as we target a different problem. Template-based layouts can be used in combination with other tiling algorithms which arrange free items from a dataset to optimize a certain metric while our approach is employed to create custom layouts where relative tile positions carry meaningful information, such as in the case of our body anatomy scenario. If multiple suitable layouts are found by the template generation algorithm as in our example in Section 5, any of the available treemapping metrics (e.g., average aspect ratio, layout distance change function) can be used to select the optimal layout. Constraints on item adjacency as in (Shneiderman and Wattenberg, 2001) will require only a minor modification of the proposed template generation algorithm and are likely to reduce its search space.

Treemaps are commonly used in combination with nodelinks such as overlaying trees or graphs. Linsen and Behrendt (Linsen and Behrendt, 2011) present a joint 3D map nodelink and treemap interactive visualization tool. While it is not an issue to support 3D layouts (one of our early prototypes featured a 3D view where a treemap was used as a “chessboard” to place 3D objects), zooming facilities in our approach are more naturally supported. We can enlarge multiple selected areas of a treemap while retaining contextual information around enlarged tiles. As pointed out in (Tu and Shen, 2008), such context-retaining multi-focus zooming is not supported by existing tools. The authors tackle this problem by constructing a dependency graph that reflects relative positions of tiles and provide an algorithm to enlarge selected areas while retaining initial relative positions of selected tiles. Non-focused items are evenly distributed in remaining area. The algorithm is non-trivial, requires solving systems of linear equations and introduces a significant computational overhead.

An extension of treemapping algorithms has been studied in the attempt to generalize them for directed acyclic graphs (DAG) (Tsiaras et al., 2009). It has been shown that such layouts are not always possible, and when they are possible, the tiling algorithm is NP complete. Heuristics have been proposed for several special cases. The DAGs can be modified by replicating nodes with two or more parents. If all such nodes are replicated, the DAG is converted to a rooted tree. What is the minimal number of replications needed to be able to build a DAG map is an open question. While the objective of this work is different from ours, the idea to encode desired characteristics of a treemap layout as constraints was inspired by this paper.

de Bono et al. (de Bono et al., 2012) describes limitations of existing treemapping tools for biomedical data visualization. This work focuses on the conceptual templates tailored for the representation of body anatomy. In our work, we introduce a generic method to build custom templates and apply them to control treemap layouts in any application domain.

Vliegen et al. (Vliegen et al., 2006) apply treemaps with various tiling algorithms for visualizing business data. The best results are obtained using the hybrid approaches. In our approach, we allow users to customize the appearance of each tile independently by assigning a template. The design guidelines for creating effective rectangular treemaps (Kong et al., 2010) can help users to define templates in our framework for the best visual perception of data depending on their quantitative characteristics. The tree-like templates introduced in our work can be targeted not at satisfying relative positioning constraints but at optimizing the layout distance change function or some other metric, including custom ones.

The complete coverage of all available area may not be required for biomedical data visualization or other application domains. In relation to this, many other variations of CSPs can be defined and solved to generate visually appealing treemap layouts. Moreover, such CSPs may require only approximate or fuzzy solution as the visualization does not always need to strictly correspond to the data attributes it displays. For example, a treemap-based tool for the visualization of mutual fund portfolios (Csallner et al., 2003) introduces a so called distorted treemap, which trades proportionality of a mapped attribute to a more inclusive visualization. The distorted treemap is a compromise between showing all the data elements and a classic treemap that preserves value proportions. Consequently, such treemap can show one more attribute than a classic treemap, though the node area is no longer proportional to the visualized attribute. This direction of work was not covered in our approach and requires further investigation.
7 Conclusions and Future Work

In this paper, we introduced a method and tools to build custom templates and apply them to customize treemaps layouts. We illustrated the application of our approach to control the positions of tiles in a sample treemap. We also presented a method for automatic generation of treemap templates for a class of problems with positional constraints. Our method generates stable layouts with an easy way to zoom in selected areas, including multi-foci case.

Our future work includes analysis of the problems of automated template generation for other classes of constraints as well as application of the presented method to the visualization of various data sets. Scalability and usability of the proposed methods will be studied more extensively. Since building custom templates for large data sets (e.g., human brain ontology) is time-consuming and requires specialized knowledge, a shared platform for template storage and reuse will be useful. We are also planning to extend our method to define templates for generating 3D images from biomedical ontologies.

REFERENCES